IDENTIFICATION OF CONTACT THERMAL RESISTANCES IN NUCLEAR REACTOR FUEL ELEMENTS. 1. ALGORITHM DEVELOPMENT

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We examine the algorithms for the determination of the contact thermal resistance between a fuel and the fuel-element shells of nuclear reactors by solving inverse problems with optimum planning of nonsteady thermal experiments. We present results from the solution of methodological examples.

INTRODUCTION

In the operation of nuclear reactor fuel elements their structural elements (fuel and shell) undergo an entire range of physicochemical and nuclear-physical transformations. The fuel volume is altered as a consequence of thermal-radiation sintering, thermal expansion and cracking; the release of gaseous fission products from the fuel and their physicochemical interaction; the mechanical interaction of the fuel with shell; the change in the dimensions of the shell as a consequence of deformation and radiation growth; the change in emissivity and similar characteristics of the fuel and of the shell. All of these processes and their influence on the readiness and reliability of the fuel elements depend on the thermal regime in which these fuel elements are operating, and this is significantly dependent on the magnitude of the contact thermal resistance (CTR) between the fuel and the shell.

Computational methods which take into account the above-enumerated processes [1], based on models of conductivity in the gap between the fuel and the fuel jacket, are in turn constructed on the basis of empirical data [2, 3]. Such models must be based on experimental studies of the magnitude of the CTR and on data with respect to changes in the latter during the process of irradiation under various regimes. For this purpose we have to measure the CTR between the fuel and the fuel-element jacket when testing is carried out in research reactors.

The methods of studying such fuel elements, in use up to the present time, in research reactors, including the post-reactor studies in hot chambers, yield exceedingly tangential information with regard to the state of the clearance and the change in the CTR during the irradiation process. The specific procedure of internal reactor measurement imposes an entire range of limitations on the methods employed, and these will lead, in particular, to the impossibility of utilizing stationary methods in the determination of the CTR. A contemporary solution for this complex problem is built on resort to the approaches and methods of identifying systems with distributed parameters. The present study is devoted to the determination of the contact thermal resistance between a fuel and its shell jacket, involving the use of algorithms based on extremum methods for the solution of inverse problems. Such a problem has been studied in considerable detail in [4]; however, the case being analyzed exhibits a number of unique features. First of all, this process should be considered in a cylindrical coordinate system. Second, the distribution of temperature in the fuel element at the initial instant of time $\tau = 0$ is determined from the solution of the corresponding stationary heat-conduction problem.

Let us examine the one-dimensional process of nonsteady heat exchange in a two-layered structural element, this process taking place over the time segment $[0, \tau_m]$. We will assume here that there exists a point of ideal contact between the layers, and that the process of heat transfer in each layer is described by a nonlinear equation of heat conduction. The layers have been fabricated of various materials and exhibit thicknesses of Δ_1 and Δ_2, respectively.

The magnitude of the CTR is unknown in the solution of the inverse problem. For its determination, in addition to the mathematical model of heat conduction, the system must include additional temperature measurements at one or several points in the space of the design being analyzed.

Questions relating to the existence of a solitary solution for the inverse problem (such as is being dealt with here) were investigated in [5], where it is demonstrated that in the one-dimensional case for the reproduction of the contact thermal resistance in a two-layered system it is sufficient, generally speaking, to carry out a nonsteady measurement of temperature at a single point in space. However, in order not to limit the algorithm for the solution of the inverse problem to utilization of only the minimum possible experimental

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information, it will be assumed in the future that the number N of heat sensors exhibits some rather arbitrary value and the solution of the inverse problem thus becomes possible with consideration of an excess in experimental data. Here we will assume that the contact thermal resistance is a function of temperature. For the sake of determinacy, let this be the temperature at the point of contact from the side of the fuel.

Proceeding as in [4], we introduce the following notation: n_1, the number of heat sensors at internal points within the fuel; n_2 represents that number within the shell jacket; X_{ji}, the coordinate for the installation of the i-th sensor in the j-th layer; $i = \overline{1, n_j}$, $j = 1, 2$. In this notation we present the measurement data in the following form:

$$T_{max}(X_{ji}, \tau) = f_{ji}(\tau), \quad i = \overline{1, n_j}, \quad j = 1, 2. \tag{1}$$

Let us then formulate the mean square functional of deviation in the theoretical temperature values (at the points at which the heat sensors were mounted) from the experimental values:

$$J = \frac{1}{2} \sum_{j=1}^{2} \sum_{i=1}^{n_j} \Big(\int_0^{\tau_m} (T_{ji}(X_{ji}, \tau) - f_{ji}(\tau))^2 d\tau + \\ + (T_{ji}(X_{ji}, 0) - f_{ji}(0))^2 \Big), \tag{2}$$

where $T_{ji}(X_{ji}, \tau)$, $i = \overline{1, n_j}$, $j = 1, 2$ are the calculated temperatures at the heat-sensor locations. To determine the unknown characteristic R(T) let us construct a procedure for the minimization of functional (2).

Let us introduce into our examination the fictitious layers whose boundaries pass through the points at which the heat sensors are installed. The original model of heat conduction can then be represented as follows:

$$C_j(T) \frac{\partial T_{ji}}{\partial \tau} = \frac{1}{x^\nu} \frac{\partial}{\partial x} \left(x^\nu \lambda_j(T) \frac{\partial T_{ji}}{\partial x} \right) + S_j(x, \tau, T_{ji}),$$

$$X_{j,i-1} < x < X_{ji}, \quad 0 < \tau \leqslant \tau_m, \quad i = \overline{1, n_j + 1}, \quad j = 1, 2, \tag{3}$$

$$X_{1,0} = l_0, \quad X_{1,n_1+1} = X_{2,0} = l_1, \quad X_{2,n_2+1} = l_2,$$

$$a_1 \lambda_1 (T_{11}(l_0, \tau)) \frac{\partial T_{11}}{\partial x}(l_0, \tau) + b_1 T_{11}(l_0, \tau) = g(\tau), \tag{4}$$

$$\frac{\partial T_{ji}}{\partial x}(X_{j,i}, \tau) = \frac{\partial T_{j,i+1}}{\partial x}(X_{ji}, \tau), \quad i = \overline{1, n_j}, \quad j = 1, 2, \tag{5}$$

$$T_{ji}(X_{ji}, \tau) = T_{j,i+1}(X_{ji}, \tau), \quad i = \overline{1, n_j}, \quad j = 1, 2, \tag{6}$$

$$\lambda_1 (T_{1,n_1+1}(l_1, \tau)) \frac{\partial T_{1,n_1+1}}{\partial x}(l_1, \tau) = \lambda_2 (T_{2,1}(l_1, \tau)) \frac{\partial T_{2,1}}{\partial x}(l_1, \tau), \tag{7}$$

$$-R(T_{1,n_1+1}(l_1, \tau)) \frac{\partial T_{1,n_1+1}(l_1, \tau)}{\partial x} = T_{1,n_1+1}(l_1, \tau) - T_{2,1}(l_1, \tau), \tag{8}$$

$$a_2 \lambda_2 (T_{2,n_2+1}(l_2, \tau)) \frac{\partial T_{2,n_2+1}}{\partial x}(l_2, \tau) + b_2 T_{2,n_2+1}(l_2, \tau) = q(\tau). \tag{9}$$

The initial distribution T(x, 0) is determined from solution of the steady boundary-value problem of heat transfer in the fuel element, this problem derived out of (3)-(9), under the assumption that $\partial T_{ji}/\partial \tau \equiv 0$, $i = \overline{1, n_j + 1}$, $j = 1, 2$.

Let us further make the assumption that the function R(T) gained some small increment $\Delta R(T)$. The temperature $T_{ji}(x, \tau)$ will then change by $\vartheta(x, \tau)$. It can be demonstrated that the variation in the temperature $\vartheta_{ji}(x, \tau)$, $i = \overline{1, n_j + 1}$, $j = 1, 2$ satisfies the following boundary-value problem:

$$C \frac{\partial \vartheta_{ji}}{\partial \tau} = \frac{1}{x^\nu} \frac{\partial}{\partial x} \left(x^\nu \lambda_j \frac{\partial \vartheta_{ji}}{\partial x} \right) + \lambda_j' \frac{\partial T_{ji}}{\partial x} \frac{\partial \vartheta_{ji}}{\partial x} + \Big(\lambda_j' \frac{\partial^2 T_{ji}}{\partial x} + \\ + \lambda_j'' \left(\frac{\partial T_{ji}}{\partial x} \right)^2 + \frac{\nu}{x} \lambda_j' \frac{\partial T_{ji}}{\partial x} + \frac{\partial S_j}{\partial T} - C_j' \frac{\partial T_{ji}}{\partial \tau} \Big) \vartheta_{ji},$$

$$X_{j,i-1} < x < X_{ji}, \quad 0 < \tau \leqslant \tau_m, \quad i = \overline{1, n_j + 1}, \quad j = 1, 2, \tag{10}$$

$$a_1\lambda_1 \frac{\partial\vartheta_{11}}{\partial x}(l_0, \tau) + \left(b_1 + a_1\lambda_1' \frac{\partial T_{11}(l_0, \tau)}{\partial x}\right)\vartheta_{11}(l_0, \tau) = 0, \tag{11}$$

$$\frac{\partial\vartheta_{ji}(X_{ji}, \tau)}{\partial x} = \frac{\partial\vartheta_{j,i+1}(X_{j,i}, \tau)}{\partial x}, \quad i = \overline{1, n_j}, \; j = 1, 2, \tag{12}$$

$$\vartheta_{ji}(X_{ji}, \tau) = \vartheta_{j,i+1}(X_{ji}, \tau), \; i = \overline{1, n_j}, \; j = 1, 2, \tag{13}$$

$$\lambda_1 \frac{\partial\vartheta_{1,n_1+1}}{\partial x}(l_1, \tau) + \lambda_1' \frac{\partial T_{1,n_1+1}(l_1, \tau)}{\partial x}\vartheta_{1,n_1+1}(l_1, \tau) = \tag{14}$$

$$= \lambda_2 \frac{\partial\vartheta_{21}(l_1, \tau)}{\partial x} + \lambda_1' \frac{\partial T_{21}(l_1, \tau)}{\partial x}\vartheta_{21}(l_1, \tau),$$

$$-\lambda_1 R \frac{\partial\vartheta_{1,n_1+1}}{\partial x}(l_1, \tau) - \left(\lambda_1 \frac{\partial T_{1,n_1+1}(l_1, \tau)}{\partial x} R' + \right. \tag{15}$$

$$\left. + R\lambda_1' \frac{\partial T_{1,n_1+1}(l_1, \tau)}{\partial x}\right)\vartheta_{1,n_1+1}(l_1, \tau) - \lambda_1 \frac{\partial T_{1,n_1+1}}{\partial x}(l_1, \tau)\Delta R =$$

$$= \vartheta_{1,n_1+1}(l_1, \tau) - \vartheta_{2,1}(l_1, \tau),$$

$$a_2\lambda_2 \frac{\partial\vartheta_{2,n_2+1}}{\partial x}(l_2, \tau) + \left(b_2 + a_2\lambda_2 \frac{\partial T_{2,n_2+1}}{\partial x}(l_2, \tau)\right)\vartheta_{2,n_2+1}(l_2, \tau) = 0. \tag{16}$$

The initial distribution $\vartheta(x, 0)$ will then also be determined from the solution of the steady boundary-value problem which can be derived if we assume that $\partial\vartheta_{ij}/\partial\tau \equiv 0$, $i = 1, n_j + 1$, $j = 1, 2$.

Using the conditions of steadiness with respect to the expansion of the Lagrange functional [4] and equating to zero the corresponding coefficients, we obtain the following conjugate boundary-value problem:

$$-C_j \frac{\partial\psi_{ji}}{\partial\tau} = \frac{1}{x^\nu}\frac{\partial}{\partial x}\left(x^\nu \lambda_j \frac{\partial\psi_{ji}}{\partial x}\right) - \left(\lambda_j' \frac{\partial T_{ji}}{\partial x} - \frac{2\nu}{x}\lambda_j\right)\frac{\partial\psi_{ji}}{\partial x} +$$

$$+ \left(\frac{\nu}{x^2}\lambda_j + \frac{\partial S_j}{\partial T}\right)\psi_{j,i}, \tag{17}$$

$$X_{j,i-1} < x < X_{ji}, \; 0 \leqslant \tau < \tau_m, \; i = \overline{1, n_j + 1}, \; j = 1, 2,$$

$$a_1\lambda_1 \frac{\partial\psi_{11}}{\partial x}(l_0, \tau) + \left(b_1 - a_1 \frac{\nu}{l_1}\lambda_1\right)\psi_{11}(l_0, \tau) = 0, \tag{18}$$

$$\lambda_j\left(\frac{\partial\psi_{ji}}{\partial x}(X_{ji}, \tau) - \frac{\partial\psi_{j,i+1}}{\partial x}(X_{ji}, \tau)\right) = T_{ji}(X_{ji}, \tau) - f_{ji}(\tau), \tag{19}$$

$$i = \overline{1, n_j}, \; j = 1, 2,$$

$$\psi_{ji}(X_{ji}, \tau) = \psi_{j,i+1}(X_{ji}, \tau), \; i = \overline{1, n_j}, \; j = 1, 2, \tag{20}$$

$$R\lambda_2 \frac{\partial\psi_{21}}{\partial x}(l_1, \tau) = \left(1 + \frac{\nu}{l_1}\lambda_2\right)\psi_{2,1}(l_1, \tau) - \psi_{1,n_1+1}(l_1, \tau), \tag{21}$$

$$\lambda_1 \frac{\partial \psi_{1,n+1}}{\partial x}(l_1, \tau) = \lambda_2 \left(\lambda_1 \frac{\partial T_{1,n_1+1}}{\partial x}(l_1, \tau) R' + 1 - \right. \tag{22}$$

$$\left. - \frac{\nu}{l_1} \lambda_1 R \right) \frac{\partial \psi_{21}}{\partial x}(l_1, \tau) - \left(\frac{\nu}{l_1} \lambda_1 \lambda_2 \frac{\partial T_{1,n_1+1}}{\partial x}(l_1, \tau) R' + \right.$$

$$\left. + \frac{\nu}{l_1} \lambda_2 - \frac{\nu}{l_1} \lambda_1 - \frac{\nu^2}{l_1^2} \lambda_1 \lambda_2 R \right) \psi_{21}(l_1, \tau),$$

$$a_2 \lambda_2 \frac{\partial \psi_{2,n_2+1}}{\partial x}(l_1, \tau) + \left(b_2 - a_2 \frac{\nu}{l_2} \lambda_2 \right) \psi_{2,n_2+1}(l_2, \tau) = 0, \tag{23}$$

$$\psi_{ji}(x, \tau_m) = 0, \; i = \overline{1, n_j+1}, \; j = 1, 2, \tag{24}$$

$$\frac{1}{x^\nu} \frac{d}{dx} \left(x^\nu \lambda_j \frac{d\Phi_{ji}}{dx}(x) \right) - \left(\lambda_j' \frac{dT_{ji}}{dx} + \frac{2\nu}{x} \lambda_j \right) \frac{d\Phi_{ji}}{dx}(x) +$$

$$+ \left(\frac{\nu}{x^2} \lambda_j + \frac{\partial S_j}{\partial T} \right) \Phi_{ji}(x) + C_j \psi_{ji}(x, 0) = 0,$$

$$X_{j,i-1} < x < X_{ji}, \; \Phi_{ji} = \Phi_{ji}(x), \; i = \overline{1, n_j+1}, \; j = 1, 2, \tag{25}$$

$$a_1 \lambda_1 \frac{d\Phi_{11}}{dx}(l_0) + \left(b_1 - a_1 \frac{\nu}{l_1} \lambda_1 \right) \Phi_{11}(l_0) = 0, \tag{26}$$

$$\frac{d\Phi_{ji}}{dx}(X_{ji}) = \frac{d\Phi_{j,i+1}}{dx}(X_{ji}), \; i = \overline{1, n_j}, \; j = 1, 2, \tag{27}$$

$$\Phi_{ji}(X_{ji}) = \Phi_{j,i+1}(X_{j,i}), \; i = \overline{1, n_j}, \; j = 1, 2, \tag{28}$$

$$R\lambda_2 \frac{d\Phi_{2,1}(l_1)}{dx} = \left(1 + \frac{\nu}{l_1} \lambda_2 R \right) \Phi_{21}(l_1) - \Phi_{1,n_1+1}(l_1), \tag{29}$$

$$\lambda_1 \frac{d\Phi_{1,n_1+1}(l_1)}{dx} = \lambda_2 \left(\lambda_1 \frac{dT_{2,1}(l_1, 0)}{dx} R' + 1 - \right.$$

$$\left. - \frac{\nu}{l_1} \lambda_1 R \right) \frac{d\Phi_{2,1}(l_1)}{dx} - \left(\frac{\nu}{l_1} \lambda_1 \lambda_2 \frac{dT_{1,n_1+1}}{dx}(l_1, 0) R' + \right.$$

$$\left. + \frac{\nu}{l_1} \lambda_2 - \frac{\nu}{l_2} \lambda_1 - \frac{\nu^2}{l_1^2} \lambda_1 \lambda_2 R \right) \Phi_{2,1}(l_1), \tag{30}$$

$$a_2 \lambda_2 \frac{d\Phi_{2,n_2+1}(l_2)}{dx} + \left(b_2 - a_2 \frac{\nu}{l_2} \lambda_2 \right) \Phi_{2,n_2+1}(l_2). \tag{31}$$

The singularity in boundary-value problem (17)-(30) lies in the fact that it represents a system of equations for two conjugate variables $\psi_{ji}(x, \tau)$ and $\Phi_{ji}(x)$, solved jointly. The variable $\Phi_{ji}(x)$ is associated with the initial distribution of variations in the temperature $\vartheta_{ji}(x, 0)$.

Using relationships (17)-(30) and (10)-(16), we can represent the variation in the minimizing functional (2) in the form

$$\delta J = \int_0^{\tau_m} \Delta R \lambda_1 \frac{\partial T_{1,n_1+1}}{\partial x}(l_1, \tau) \left(\lambda_2 \frac{\partial \psi_{21}(l_1, \tau)}{\partial x} - \frac{\nu}{l_1} \psi_{21}(l_1, \tau) \right) d\tau +$$

$$+ \lambda_1 \frac{\partial T_{1,n_1+1}}{\partial x}(l_1, 0) \left(\lambda_2 \frac{d\Phi_{21}}{dx}(l_1) - \frac{\nu}{l_1} \lambda_2 \Phi_{21}(l_1) \right) \Delta R. \tag{32}$$

We will approximate the unknown function R(T) by the expression

$$R(T) \simeq \sum_{k=1}^{m} R_k \varphi_k(T), \tag{33}$$

where $\varphi_k(T)$, $k = \overline{1, m}$ is a system of basis functions.

The increment in this function is written as follows:

$$\Delta R(T) = \sum_{k=1}^{m} \Delta R_k \varphi_k(T). \tag{34}$$

Utilizing relationships (32) and (33), we can represent the formula for the components of the gradient in the nonclosure functional (2) with respect to the components R_k, $k = \overline{1, m}$ in the form

$$J'_{R_k} = \int_0^{\tau_m} \lambda_1 \frac{\partial T_{1,n_1+1}(l_1, \tau)}{\partial x} \left(\lambda_2 \frac{\partial T_{21}}{\partial x}(l_1, \tau) - \frac{\nu}{l_1} \lambda_2 \psi_{21}(l_1, \tau) \right) \varphi_k(T_{1,n_1+1}(l_1, (\tau)))\, d\tau + \lambda_1 \frac{dT_{1,n_1+1}}{dx}(l_1, 0) \left(\lambda_2 \frac{d\Phi_{21}}{dx}(l_1) - \frac{\nu}{l_1} \lambda_2 \Phi_{21}(l_1) \right) \varphi_k(T_{1,n_1+1}(l_1, 0)). \tag{35}$$

If we know the gradient of the nonclosure functional, we can construct an iteration algorithm for the sequential refinement of the sought function R(T) on the basis of the method of conjugate gradients:

$$\overline{R}^{s+1} = \overline{R}^s - \gamma^s \overline{G}^s,$$

where

$$\overline{R} = \{R_k\}_1^m,\ \overline{G}^s = (\overline{J}')^s + \beta^s \overline{G}^{s-1};\ \overline{G} = \{G_k\}_1^m;\ \overline{J}' = \{J'_{R_k}\}_1^m;$$
$$\beta^0 = 0;\ \beta^s = < (\overline{J}')^s - (\overline{J}')^{s-1};\ (\overline{J}')^s >_{R^m} / \| (\overline{J}')^{s-1} \|_{R^m}. \tag{36}$$

For the descent parameter we can use the linear estimate [6]

$$\gamma = - \frac{\sum_{j=1}^{2} \sum_{i=1}^{nj} \int_0^{\tau_m} (T_{ji}(X_{ji}, \tau) - f_{ji}(\tau))\, \vartheta_{ji}(X_{ji}, \tau)\, d\tau}{\sum_{j=1}^{2} \sum_{i=1}^{nj} \int_0^{\tau_m} (\vartheta_{ji}(X_{ji}, \tau))^2\, d\tau} - \frac{\sum_{j=1}^{2} \sum_{i=1}^{nj} (T_{ji}(X_{ji}, 0) - f_{ji}(0))\, \vartheta_{ji}(X_{ji}, 0)}{\sum_{j=1}^{2} \sum_{i=1}^{nj} (\vartheta_{ji}(X_{ji}, 0))^2}, \tag{37}$$

where the function $\vartheta_{ji}(x, \tau)$ satisfies boundary-value problem (10)-(16) when $\Delta R_k = \sum_{k=1}^{m} G_k \varphi_k(T)$.

We should note that the algorithm for the solution of the inverse problems R = const can be significantly simplified by carrying out the process of successive approximations without calculating the gradient of the nonclosure functional [6]. In this case, for each iteration we have the correction factor ΔR. We can demonstrate that this correction factor is calculated from formula (36), where the functions $\vartheta_{ji}(x, \tau)$ are determined for $\bar{G} = 1$.

This algorithm for the solution of the inverse problem has been attained in the form of a program complex which represents a modification and expansion of the complex described in [4].

In the parametric identification of the heat-exchange processes the achievement of maximum reliability and accuracy for the derived results is an important problem. An effective means of solving this problem is based on the utilization of approaches and methods from the theory of experimental planning [7]. In this case, an a priori estimate of the quality of the conducted experiments is undertaken and a preliminary search is conducted for the experimental conditions under which the chosen quality criterion reaches its maximum value.

With regard to the determination of the contact thermal resistances, we are confronted, in particular, with the problem of optimizing the temperature measurement scheme. It is necessary to choose a measurement scheme such that the maximum accuracy in the solution of the inverse problem is achieved. In carrying out direct concentrated dynamic measurements of temperature at N points in space with coordinates X_i, $i = \overline{1, N}$ during the experiment, we can represent the measurement plan by means of the following vector [7]:

$$\xi = \{N, \overline{X}\},\ \overline{X} = \{X_i\}_1^N. \tag{38}$$

Selection of the measurement plan that is optimum in this sense reduces to the solution of the extremum problem

$$\xi_0 = \operatorname{Arg}\max_{\xi \in \Xi} \psi(M(\xi)), \tag{39}$$

where $\psi(M(\xi))$ is the selected criterion of quality, which can be constructed on the basis of the information matrix [7]

$$M(\xi) = \{\Phi_{jk}\}, \; j, \; k = \overline{1, m}, \tag{40}$$

where $\Phi_{jk} = \frac{1}{N} \sum_{i=1}^{N} \int_0^{\tau_m} \vartheta_j(X_i, \tau)\vartheta_k(X_j, \tau)d\tau$; Ξ is the set of permissible measurement plans; $\vartheta_k(x, \tau) = \partial T/\partial R_k$, $k = \overline{1, m}$ are the sensitivity functions, R_k is the vector component of the unknown functions, formed subsequent to the parametrization of an unknown function R(T) in the form of (32). In particular, we can use the eigenvalues of the information matrix $k = \overline{1, m}$, and as the criteria to analyze the following quantities [7]: the reciprocal of conditionality $1/C(M) = \sqrt{\mu_{min}/\mu_{max}}$, the square root of the minimum eigenvalue $\sqrt{\mu_{min}}$, the determinant of the informtion matrix $\det M = \prod_{k=1}^{m} \mu_k$ etc.

The permissible plan set Ξ is formed on the basis of results obtained from an investigation into the conditions of single-valued solvability for the planned inverse problem. This set is defined as follows:

$$\Xi = \{(N, X) : N \geqslant N_{min}, \; X_i \in [l_0, \; l_2], \; i = \overline{1, N,}\}, \tag{41}$$

where N_{min} is the minimum necessary number of heat sensors at which singularity in the solution of the inverse problem is achieved. In the case under consideration $N_{min} = 1$ [5].

Owing to the nonlinearity of boundary-value problem (3)-(9) the sensitivity functions $\vartheta_k(x, \tau)$, $k = \overline{1, m}$ and, consequently, the normalized information matrix M depend on the vector of the parameters $\{R_k\}_1^m$ sought in the inverse problem. Only local-optimum planning of measurements is therefore possible by resort to a priori information on vector $\bar{R}$ [7].

An iteration computational algorithm for the solution of one-dimensional problems of optimum planning is offered in [7] and it is based on the numerical solution of the corresponding boundary-value problems and the finding of an optimum measurement plan on a fixed finite-difference grid.

For purposes of analyzing the effect of various factors on the final result in the experimental-theoretical determination of the contact thermal resistances we conducted computational experiments simulating the complex procedure of identification.

We analyzed contact heat exchange between the fuel and the external shell jacket of a cylindrical rod heat-evolving element. The mathematical model of the analyzed heat-exchange process has the form of (3)-(9) with consideration given to the fact that

$$S_1(x, \; \tau, \; T_1) = q_v(x, \; \tau), \tag{42}$$

$$S_2(x, \; \tau, \; T_2) = 0, \tag{43}$$

$$a_2 = -1, \; b_1 = 0, \; g(\tau) = 0, \tag{44}$$

$$a_2 = -1, \; b_2 = \alpha(\tau), \; q(\tau) = -\alpha(\tau) T_e(\tau). \tag{45}$$

Here $q_v(x, \tau)$, $\alpha(\tau)$, $T_e(\tau)$, $C_1(T_1)$, $C_2(T_2)$, $\lambda_1(T_1)$, and $\lambda_2(T_2)$ are known functions. The nonsteadiness of the fuel-element thermal regime is governed by the change of energy release $q_v(x, \tau)$ over time in the fuel or by the conditions of heat exchange at the external boundary $\alpha(\tau)$, $T_2(\tau)$. The thermophysical characteristics of the fuel and material and of the shell, as used in the computations, are presented in [8, 9]. The release of energy in the fuel was calculated so as to account for radial nonuniformity [10] and the integral release of heat, determined by means of thermal neutron detectors. The remaining initial data were assumed to be the following [11]: $l_0 = 0.9$ mm, $l_1 = 2.975$ mm, $l_2 = 3.45$ mm, $\alpha = 1.36 \cdot 10^4$ W/(m$^2\cdot$K), $T_e = 773$ K.

It was the procedure of identifying R = const that was initially simulated. Figure 1 shows the results from the solution of local-optimum measurement planning problems for three a priori given values of the unknown parameter R under the assumption that the measurements are equally accurate (the dispersion in the measurement errors was assumed to be $\sigma^2 = 1$ for the entire range of measured temperatures). In this case the information matrix degenerates into the scalar quantity M, which serves as the criterion of optimality. The derived results show that the most rational solution for the inverse problem from the standpoint of accuracy is the positioning of the heat sensor in the fuel, in particular at the surface of the inside channel where $X = l_0$. It is precisely at these values of X that we achieve the maximum value for the criterion M.

To test and confirm the measurement planning results presented here for temperature, we undertook a parametric analysis of the accuracy of the solution for the inverse problem of restoring the parameter R. The solution of the model inverse problem

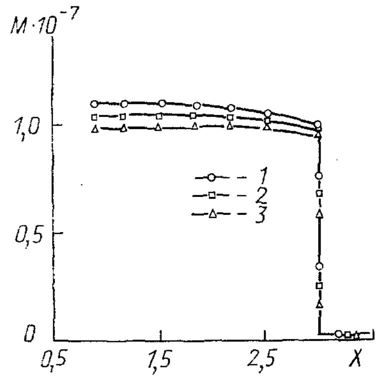

Fig. 1

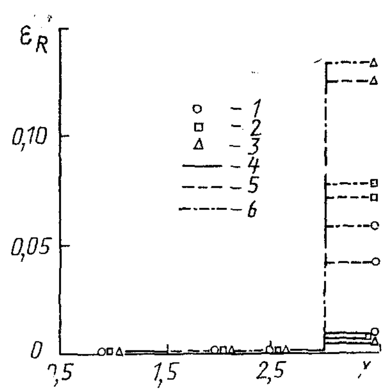

Fig. 2

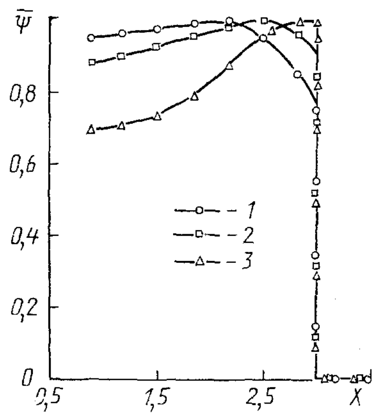

Fig. 3

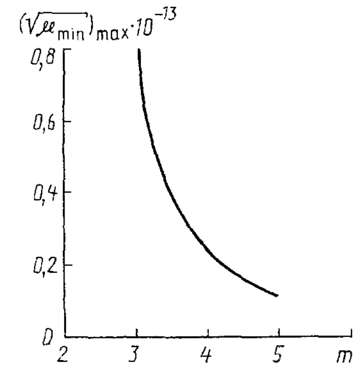

Fig. 4

Fig. 1. Change in the planning criterion M as a function of the heat-sensor position coordinate X for various values of R, K/(W/m^2): 1) R = 0.125·10^{-3}; 2) 0.2·10^{-3} 3) 0.3·10^{-3}. X, mm.

Fig. 2. The error ε_R in the reproduction of R for various heat-sensor positions and variation in the values of Δ_m and R, K/(W/m^2): 1) Δ_m = 0; 2) 0.03; 3) 0.05; 4) R = 0.125·10^3; 5) 0.2·10^{-3}; 6) 0.3·10^{-3}.

Fig. 3. Change in the relative criterion $\bar{\psi}$ as a function of the heat-sensor position coordinate X for various numbers of approximation parameters: 1) m = 3; 2) 4; 3) 5.

Fig. 4. Maximum value of the optimality criterion $(\sqrt{\mu_{min}})_{max}$ as a function of the number of approximation parameters.

was reproduced both for "exact" values of the temperature "measured" in the dynamic regime, and with consideration given to the possible random measurement errors. The errors were simulated by means of a random number generator. The results from the computational experiment in the form of a relationship between the relative error in the solution of the inverse problem and the location coordinate X of the heat sensor are shown in Fig. 2 ($\varepsilon_R = |R - R^{exact}|/R^{exact}$).

The problem of identifying the contact thermal resistance as a function of temperature was then analyzed. For the a priori information regarding the unknown characteristic R(T) we specified a constant value for R^{exact} = 0.2·10^{-4} (K·m^2)/W. The remaining initial data were the same as before. The results obtained in our search for the optimum positioning of a single heat sensor while varying the number of approximation parameters are shown in Fig. 3, where we find the change in the relative optimality criterion $\bar{\psi} = \sqrt{\mu}/(\sqrt{\mu_{min}})_{max}$ as a function of the location coordinate of the heat sensor. We see that when m = 3 the sensor should be positioned inside the fuel, and when m is increased the point of optimum sensor location is displaced toward the boundary between the fuel and the protective jacket, but it nevertheless remains inside the fuel. The change in the maximum value of the criterion $(\sqrt{\mu_{min}})_{max}$ with an increase in m is shown in Fig. 4 and suggests intensive impairment in the conditionality of the inverse problem with an increase in m.

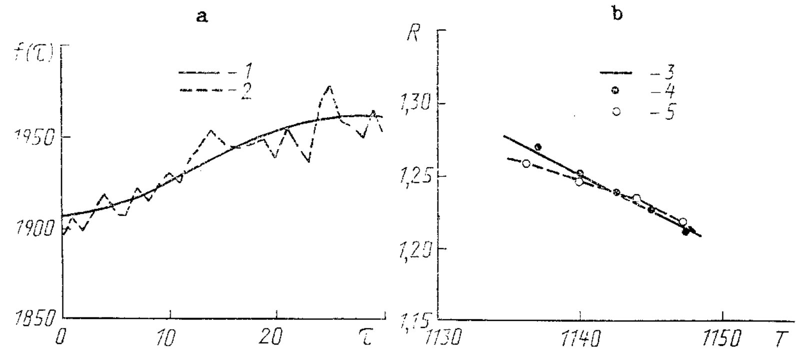

Fig. 5. Change in temperature at the location of the heat sensor (a) and results from the solution of the inverse problem (b): 1) exact measurement of f(τ), K; 2) measurements with errors; 3) specified value of R, K/(W/m²); 4) solution with utilization of exact measurements of f(τ); 5) solution with measurement containing errors. τ, sec; T, K.

As an example to confirm the planning results we undertook to solve the model inverse problem of determining the function R(T) given in the form of a linear function of temperature. The number of approximation segments in this unknown function was assumed to be equal to three. Figure 5 shows the results from the solution of the inverse problem.

CONCLUSIONS

The purpose of this study is to describe the developed algorithm by means of which to analyze and process the data of the nonsteady thermal experiment in the identification of the CTR between the fuel and the nuclear-reactor fuel-element shell jacket. An iteration algorithm has been constructed for the numerical solution of the inverse heat-conduction problem based on minimization by gradient methods of the nonclosure functional. The gradient of the functional is calculated in conjunction with the solution of the derived boundary-value problem for the conjugate variable. The iteration process is stopped on the basis of the nonclosure criterion.

We present a formulation of the problem of optimizing the temperature measurement scheme. An algorithm for its numerical solution is constructed.

By means of the developed algorithms for the solution of the primary mathematical problems of identifying the CTR and the corresponding program we have conducted computational experiments to simulate the complex procedure of identification. The derived results confirm the utility of the mathematical apparatus created here, and the possibility of its effective use in carrying out experimental-theoretical research into contact heat exchange in fuel elements.

The method of carrying out test-stand experiments, the result from the treatment and analysis of experimental data, these will all be dealt with in a subsequent portion of the present study.

NOTATION

T, temperature; τ, time; x, space coordinate; N, number of heat sensors; f(τ), measured temperature values; X, coordinate of heat-sensor location; C(T), volumetric heat capacity; λ(τ), thermal conductivity; S(x, τ, T), heat source; g(τ) and q(τ), external thermal effect; R(T), contact thermal resistance; Δ, thickness of layer; J, nonclosure functional; ψ, conjugate variable; ϑ_{ji}, temperature variation; j, heat-sensor number; i, layer number; ν, coordinate system index.

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AN ALGORITHM FOR A NUMERICAL STUDY OF THE HYDRODYNAMICS AND EXCHANGE OF HEAT IN A TWISTED CHANNEL OF COMPLEX CROSS SECTION ON THE BASIS OF THE FINITE-ELEMENT METHOD

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UDC 532.5:536.24

We propose the approximation and an algorithm for a numerical solution of the heat and mass transfer problem in a twisted channel of complex transverse cross section.

One method of intensifying the processes of heat and mass transfer in tubes and channels involves the twisting of the heat-carrying coolant in the inlet section with the aid of various turbulators. The resulting rotation of the flow in this case is retained over some initial segment of the channel, the length of this segment dependent on the shape of the transverse cross section, the initial kinetic energy of fluid rotation, its physical properties, the condition of the wall surfaces, etc. In a number of cases, the effect of these factors is so significant that the method by means of which the flow is twisted at the inlet to the channel becomes almost ineffective as a consequence of the rise in the level of energy expended on the pumping of the coolant and on its twisting, relative to the increased release of heat.

An alternate method of setting the flow in the rotation involves the use of channels with an extended twist (twisted channels), producing the effect of fluid rotation over the entire channel length. The efficiency achieved by such a method of intensifying the transfer processes, based on a series of complex experimental investigations into the hydrodynamics and heat and mass exchange in equipment with coiled tubes of oval and trefoil profiles, is demonstrated in [1-3]. At the same time, the literature contains no description of systematic approaches to the numerical modeling of transfer processes in twisted channels of complex cross section. A number of computational results [4, 5] have been obtained by the method of finite differences, while the computational algorithms constructed on the basis of this method have found their application limited to channels of classical cross-sectional shape (circular, rectangular).

The present study is devoted to an outline of the computational algorithm with which to study the processes of transfer in channels of complex cross section with longitudinal twist, using the finite-element method (FEM). By using this method is becomes possible significantly to expand the spectrum of modeled processes from the standpoint of diversity in the geometric shapes of the channel cross sections, to formulate a unique approach in the construction of discrete transfer-equation analogs, etc.

The processes of hydrodynamics and heat exchange in a channel of arbitrary transverse cross section for the case of the laminar flow of a viscous incompressible fluid, the absence of heat sources or sinks in the flow, and negligibly low viscous dissipation, are described by the following system of equations:

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